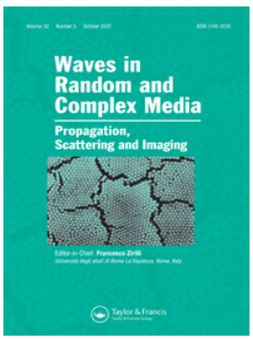

Waves in Random and Complex Media

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/twrm20

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To cite this article: Thongchai Botmart, Syed Zahir Hussain Shah, Zulqurnain Sabir, Wajaree weera, R. Sadat, Mohamed R. Ali & Wael Al-Kouz (2022): The inclination of magnetic dipole effect and nanoscale exchange of heat of the Cross nanofluid, Waves in Random and Complex Media, DOI: 10.1080/17455030.2022.2128225

To link to this article: https://doi.org/10.1080/17455030.2022.2128225

Published online: 29 Sep 2022.

WAVES IN RANDOM AND COMPLEX MEDIA
https://doi.org/10.1080/17455030.2022.2128225

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The inclination of magnetic dipole effect and nanoscale exchange of heat of the Cross nanofluid

Thongchai Botmart[a], Syed Zahir Hussain Shah[b], Zulqurnain Sabir[b], Wajaree weera[a], R. Sadat[c], Mohamed R. Ali [d,e] and Wael Al-Kouz[f]

[a]Faculty of Science, Department of Mathematics, Khon Kaen University, Khon Kaen, Thailand; [b]Department of Mathematics and Statistics, Hazara University, Mansehra, Pakistan; [c]Faculty of Engineering, Department of Mathematics, Zagazig University, Zagazig, Egypt; [d]Faculty of Engineering and Technology, Future University, Cairo, Egypt; [e]Basic Engineering Mathematics Department, Benha Faculty of Engineering, Benha University, Egypt; [f]College of Engineering and Technology, American University of the Middle East, Egaila, Kuwait

ABSTRACT

Nanoscale energy exchange and movement of fluid by Lorentz force is the most recent problem that has a strong connection with applied mathematics due to the consumption of energy. Imposing the inclination of Magnetic dipole in the fluid has abundant applications in magnetic drug engineering, astrophysics, sensors, geophysics, cosmology, and targeting. This study is associated with the exchange of energy under the influence of the inclination of Magnetic dipole with cylinder geometry. The Cross nanofluid mathematical model is launched to carry the nanoscale energy exchange with temperature-dependent thermal conductivity. The permeable cylinder geometry along with the different effects is provided during the expanding/contracting cylinder. Whereas Cross fluid helps to investigate the problem in shear thinning/thickening regions. The numerical representations of the problem are provided via the BVP4C scheme.

ARTICLE HISTORY
Received 8 December 2021
Accepted 19 September 2022

KEYWORDS
Cross nanofluid; heat transport at nanoscale; Lorentz force effect; temperature-dependent conductivity; BVP4C

Nomenclature

b_0	Constant of dimension length. t
$K(T)$	Variable thermal conductivity
β	Constant of contraction – expansion
$U_w(x,t)$	Initial velocity
$U_e(x,t)$	Free stream velocity
T_w, C_w	Temperature and concentration of the cylinder
T_∞, C_∞	Temperature and concentration of the cylinder
a, c	Constant of dimension $(time)^{-1}$
τ	Cauchy stress tensor
k	Thermal conductivity
x, r	Space variable
s	Suction parameter

CONTACT Mohamed R. Ali mohamed.reda@bhit.bu.edu.eg

k_∞	Conductivity away for
Pr	Prandtl number
ξ	Chemical rate reaction parameter
λ	Velocity ratio parameter
μ_0	zero shear rate viscosity
A_1	First Rivilin Erickson's tensor
μ_∞	Infinite shear rate viscosity
p	Pressure
I	Identity tensor
Γ	Relaxation time constant
n	Power law index
Sc	Schmidt number
μ	Viscosity
q_w	Wall shear stress
τ_{rx}	Heat flux
M	Magnetic parameter
$u,\ v$	Velocity component
c_f	Skin friction
B_0	Magnetic field strength constant
$\dot{\gamma}$	Shear strain
B	Magnetic field strength
c_p	Specific heat
D	Solute Diffusivity
R_r	Chemical reaction parameter
σ	Reaction rate parameter
ρ	Density
C	Concentration profile
T	Temperature profile
V	Velocity profile
θ_w	Temperature ratio parameter
A	Unsteadiness parameter
We	Weissenberg
α_m	Thermal diffusivity
Re	Reynold number
Nu	Nusselt number

1. Introduction

The technology and thermal engineering fields have been facing energy consumption problems for many decades. These energy consumptions can be handled to increase the thermal conductivity of nanoliquids. Nanoscale heat transport is very effective for this purpose by considering nanofluids. The Nanoscale process of heat transport is utilized for minimizing emerging energy issues. The formation of fluid is done by imposing the bombardment of solid particles. Consequently, the obtained nanoparticles have less than 100 nm dimension are called nanofluids. Nanofluid is a stream of all nanoparticles and boosts energy performance by increasing thermal conductivity. To make better energy

performance by increasing the thermal conductivity is considered an advance and valid procedure. Recently, many scholars worked on nanofluid and nanoscale heat transport. Viscoelastic nanofluid with imposed external buoyancy forces was explored by Waqas et al. [1]. The flow optimization with SiC/EG-water nanofluids and heat transfer analysis was made by Yang et al. [2]. The prediction of heat transport using thermal conductivity was explored by Komeilibirjandi et al. [3]. Yang et al. [4] addressed the air purification and heat recovery phenomenon with the highlights of nanofluids. Puneeth et al. [5] considered an effective hybrid nanofluid with the effects of bioconvection and radiation of heterogeneous – homogeneous chemical reactions.

The association of the magnetic fields and their related properties based on their fluid mechanism is known as magnetohydrodynamics (MHD). A magnetic field creates electricity and force, which, combining current and magnetic effect, is called Lorentz force. The phenomenon of MHD has various real-life applications, such as electric power generation systems, development of high-efficiency, contamination and determination of ECG signal, magneto resonance imaging (MRI), incrementing the amplitude of the T wave, molten metal, metal transformation procedure, nanofluid pumping, solid-state mixtures, pumping of seawater, fluid pumping, the industry of crystal growing, industry of polymers, MHD accelerations, MHD thrusters, MHD generator/motor, and flow control around hypersonic [6–17]. The theory related to MHD depicts the influence of magnetic effects. The Lorentz force is generated due to the effects of fluid motion. Keeping in view of the vital effects of the magnetic dipole on fluid motion, many scholars contributed by placing perpendicular/inclined magnetic dipole over fluid flow. The gyrotactic motile microorganism with Cross nanofluid and unsteady MHD aspect of flow over the geometry of a heated sheet was discovered by Reddy et al. [18]. Bioconvection, nonlinear thermal radiation, and melting process over wedge geometry were depicted by Waqas et al. [19]. Thermal conductivity performance to boost the heat transfer and magneto effect with cylinder-constructed geometry was numerically achieved by Imran et al. [20]. Furthermore, Noor et al. [21,22] worked on MHD viscous flow over a linearly stretching sheet embedded in a non-Darcian porous medium and they presented the numerical results with attached physical parameters.

Furthermore, the saturated flow of Cross nanofluid and its dynamics of multiple solutions attached facts of heat source effects and cross-diffusion were found numerical by [21,22].

Chemical reactions diverted human's life from tease to ease. It made human life very easy and there is no world progress without chemical reactions. There is a need for minimum energy, which activates the chemical reaction known as activation energy. A chemical reaction, which remains fixed during the process, is called a homogeneous chemical process. On the other hand, a chemical process is able to occur everywhere, which is referred to a heterogeneous chemical process. Physically, there are two types of chemical processes, one is a constructive chemical process and the other one is a destructive process. In fluid mechanism, chemical reaction and Activation energy are utilized to judge the mass transport of fluid. According to many investigations, the chemical process plays a vital role in the mass transport of fluid. Xiong et al. [23] worked on the dynamics of Cross nanofluid and its multiple solutions with the geometry of thin needlepoint. The same work related to the dual solution with the addition of external buoyancy force was pointed out by Hafeez et al. [24]. Chemical reaction process and heat sink/source effect of nanofluid and their numerical outcomes are displayed in the scientific community by Li et al. [25]. Chemical reactions help

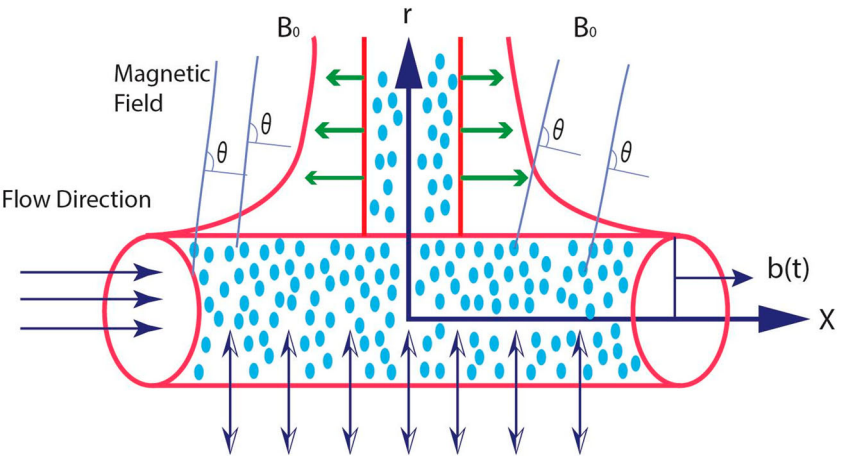


Figure 1. The geometry of flow in the permeable cylinder.

with autocatalytic and heterogeneous catalysis over the physical shape of a rotating disk as shown by Yu et al. [26]. Furthermore, Yu et al. [26] considered Ostwald-de-Waele nanofluid over rotating disk geometry with heterogeneous catalysis. MHD hybrid nanofluid with the feature of multiple slips with the facts of the autocatalytic chemical process along with the Cattaneo – Christov heat flux was discovered by Gul et al. [27] and others [28–33].

This article presents the numerical judgment of energy exchange and inclination of magnetic dipole effects on time-dependent Cross nanofluid with cylinder geometry. The viscosity of the Cross nanofluid model is utilized to obtain the numerical results of nanoscale energy exchange and temperature-dependent variable thermal conductivity. Permeable cylinder geometry represents the effect of several parameters during the expanding/contracting cylinder and the Cross fluid helps to investigate the problem in shear thinning/thickening region.

2. Geometry of flow in the permeable cylinder and problem formulation

The flow is passing through the permeable cylinder with the competence of extraction/expansion, whose time dependent radius is $b(t) = (1 - \beta t)^{1/2} b_0$, where b_0 is a positive constant and β is a constant of contraction/expansion with positive and negative values. $U_w(x,t) = \frac{2cx}{1-\beta t}$ in mathematics is called time-dependent stretching/shrinking velocity and $U_e(x,t) = \frac{2ax}{1-\beta t}$ is the free stream velocity. The stagnation point is also considered in the cylinder geometry at $r = b_0$, with $x = 0$. The inclined magnetic field $B(t) = \frac{\beta_0}{\sqrt{(1-\beta t)}}$ is imposed in the radial direction of the cylinder. Passing flow through the cylinder is considered 2D and incompressible. Surface temperature, surface concentration, ambience temperature, and ambience concentration of cylinder denoted by T_w, C_w, T_∞, C_∞ and $T_w > T_\infty$. Figure 1

A tensor of the Cross model and all its related mathematics are listed in Equations (1) and (2).

$$\tau = -pI + \mu(\dot{\gamma})A_1.$$

$$\mu(\dot{\gamma}) = \left[\mu_\infty + \frac{\mu_o - \mu_\infty}{1 + (\Gamma\,\dot{\gamma})^n}\right]. \tag{1}$$

$$\left(\dot{\gamma} = \sqrt{\frac{1}{2}tr(A_1)^2}, \qquad A_1 = (\nabla V) + (\nabla V)^T\right). \tag{2}$$

The velocity assumption is taken as

$$V = [u(r,x,t),\, v(r,x,t), 0], T = T(r,x,t), C = C(r,x,t). \tag{3}$$

The boundary layer analysis (BLA) and formulated problem take the following mathematical form:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial x} = 0, \tag{4}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = U_e\frac{\partial U_e}{dx} + \frac{\upsilon}{r}\frac{\partial u}{\partial r}\left[\frac{1}{\left[\left(1+\Gamma\frac{\partial u}{\partial r}\right)^n\right]}\right] + \frac{\partial U_e}{dt},$$

$$+ \upsilon\frac{\partial}{\partial r}\left[\frac{\frac{\partial u}{\partial r}}{\left[\left(1+\Gamma\frac{\partial u}{\partial r}\right)^n\right]}\right] + \frac{\sigma B^2(t)}{\rho}\sin^2(\varpi)(u - U_e), \tag{5}$$

$$\left[\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial r} + u\frac{\partial T}{\partial x}\right] = k\frac{1}{r}\frac{\partial}{\partial r}\left[k(T)\,r\frac{\partial T}{\partial r}\right], \tag{6}$$

$$\left(\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial r} + u\frac{\partial C}{\partial x}\right) = D\left[\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r}\right], \tag{7}$$

The associated boundary conditions (BCs) are

$$u = U_w(x,t) = \frac{2cx}{1-\beta t}, v = V_w(t) = -\frac{ab_0 s}{\sqrt{1-\beta t}},$$

$$T = T_w, C = C_w \text{ at } r = b(t), \tag{8}$$

$$r \to \infty,\, C \to C_\infty,\; T \to T_\infty\;,\; u \to U_e(x,t), \tag{9}$$

$$K(T) = k_\infty\left(1 + \varepsilon\left(\frac{T - T_\infty}{T_w - T_\infty}\right)\right), \tag{10}$$

$$u = \frac{2ax}{(1-\beta t)}f'(\eta)\,, v = -\frac{ab_0}{(1-\beta t)}\frac{f(\eta)}{\sqrt{\eta}}\;\;,\;\; \eta = \left(\frac{a_0}{r}\right)^{-2}(1-\beta t)^{-1}.$$

$$\theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty},\, \phi(\eta) = \frac{C - C_\infty}{C_f - C_\infty}, \tag{11}$$

$$\eta(\,1 + (1-n)We^n(f'')n)f''\prime + (2 + (1-n)We^n(f'')n)\tfrac{1}{2}f''$$

$$+ Re(ff'' - f'^2 + 1)(1 + We^n(f'')n)^2 - A(\eta f'' + f' - 1)(1 + We^n(f'')n)^2$$

$$- M^2\sin^2(\varpi)Re(1 + We^n(f'')n)^2(f' - 1) = 0, \tag{12}$$

$$(1 + \varepsilon\theta)\eta\theta'' + (1 + \varepsilon\theta)\theta' + \varepsilon\eta\theta' 2 - \mathrm{Pr}A\eta\theta' - \mathrm{Pr}Ref\theta' = 0, \tag{13}$$

$$\eta\phi'' + \phi' - AScn\phi' + ReScf'\phi' - ReScB^*\phi = 0, \tag{14}$$

The BCs related to the above system of ODEs are given as follows:

$$\begin{aligned}&\phi(1)=1, f^{'}(1)=\lambda, f(1)=s, \theta(1)=1,\\ &\quad as\ \eta\to\infty,\ \theta(\eta)\to 0, f^{'}(\eta)\to 1,\ \ \phi(\eta)\to 0,\end{aligned} \tag{15}$$

$$\begin{aligned}Re&=\frac{ab_0^2}{2\nu},\ A=\frac{\beta b_0^2}{4\nu},\ Sc=\frac{\nu}{D}, \Pr=\frac{\nu}{\alpha_m},\\ We&=\frac{4r\Gamma U_e}{b_0(1-\beta t)},\ \xi=\frac{R_r(1-\beta t)}{2a}, M^2=\frac{\sigma\beta_0^2}{2\rho a}, \theta_w\\ &=\frac{T_w}{T_\infty}, \lambda=\frac{c}{a},\end{aligned} \tag{16}$$

Flow mechanism and transformation of heat are the Nusselt numbers *Nu* and the local skin friction coefficient C_f is given as follows:

$$Nu=\frac{b(t)q_w|_{r=b(t)}}{2k(T_w-T_\infty)}\ and\ C_f=\frac{\tau_{rx}|_{r=b(t)}}{\frac{1}{2}\rho U_e^2}, \tag{17}$$

q_w is the wall heat flux for shear stress, τ_{xx} represents the wall shear stress, and both are defined as follows:

$$\tau_{xx}=\mu_0\frac{\partial u}{\partial r}\left[\frac{1}{1+\Gamma^n\left(\frac{\partial u}{\partial r}\right)^n}\right]_{r=b(t)}, q_w=-k\frac{\partial T}{\partial r}|_{r=b(t)}, \tag{18}$$

The Nusselt number *Nu* and the local skin friction coefficient C_f taken the following form:

$$\begin{aligned}Nu&=-\theta'(1)\\ C_fRe\frac{x}{b(t)}&=f''(1)\left[\frac{1}{1+We^n(f''(1))^n}\right],\end{aligned} \tag{19}$$

3. Methodology for Bvp4c

Nonlinear systems of ODEs attached with boundary conditions (BCs) are shot down into the system of linear equations with initial conditions (ICs). The methodology is illustrated as under

$$F=l_1,\ l_1'=l_2,\ l_2'=l_3,$$

$$y'_3=\frac{1}{\eta(1+(1-n)We^n(l_3)^n)}\times\begin{bmatrix}-(2+(1-n)We^n(l_3)^n)\frac{1}{2}l_3\\ -Re(l_1l_3-{l_2}^2+1)(1+We^n(l_3)^n)^2\\ +A(\eta l_3+l_2-1)(1+We^n(l_3)^n)^2\\ +M^2\sin^2(\varpi)Re(1+We^n(l_3)^n)^2(l_2-1)=0,\end{bmatrix}$$

$$l_4=\theta,\ {l_4}'=l_5$$

$${l_5}'=\frac{1}{(1+\varepsilon l_4)\eta}\left[\Pr A\eta\, l_5+\Pr Rel_1l_5-(1+\varepsilon l_4)l_5-\varepsilon\eta(l_5)^2\right]$$

Furthermore, the concentration form is written as follows:

$$l_6 = \phi,\ l_6' = l_7$$

$$y_7' = \frac{1}{\eta}[ASc\eta l_7 - l_7 - ReScl_2 l_7 + ReScBl_6]$$

Now the initial conditions are written as follows:

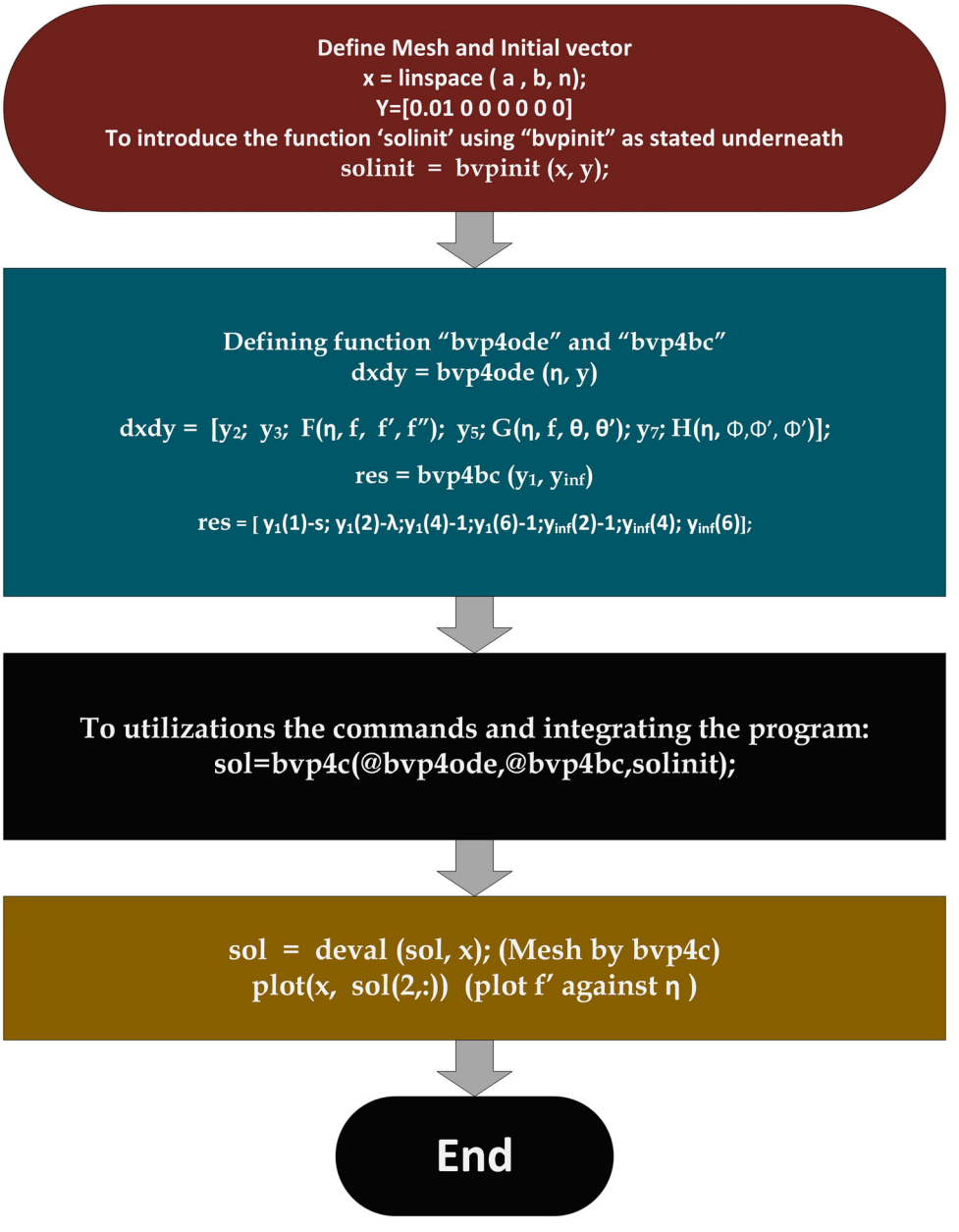


Chart 1. Methodology.

$$l_6(1) = 1, l_2(1) = \lambda, l_1(1) = s, l_4(1) = 1,$$
$$as\ \eta \to \infty,\ l_4(\eta) \to 0, l_2(\eta) \to 1,\ \ l_6(\eta) \to 0,$$

4. Methodology for Keller Box

Chart 1 and 2

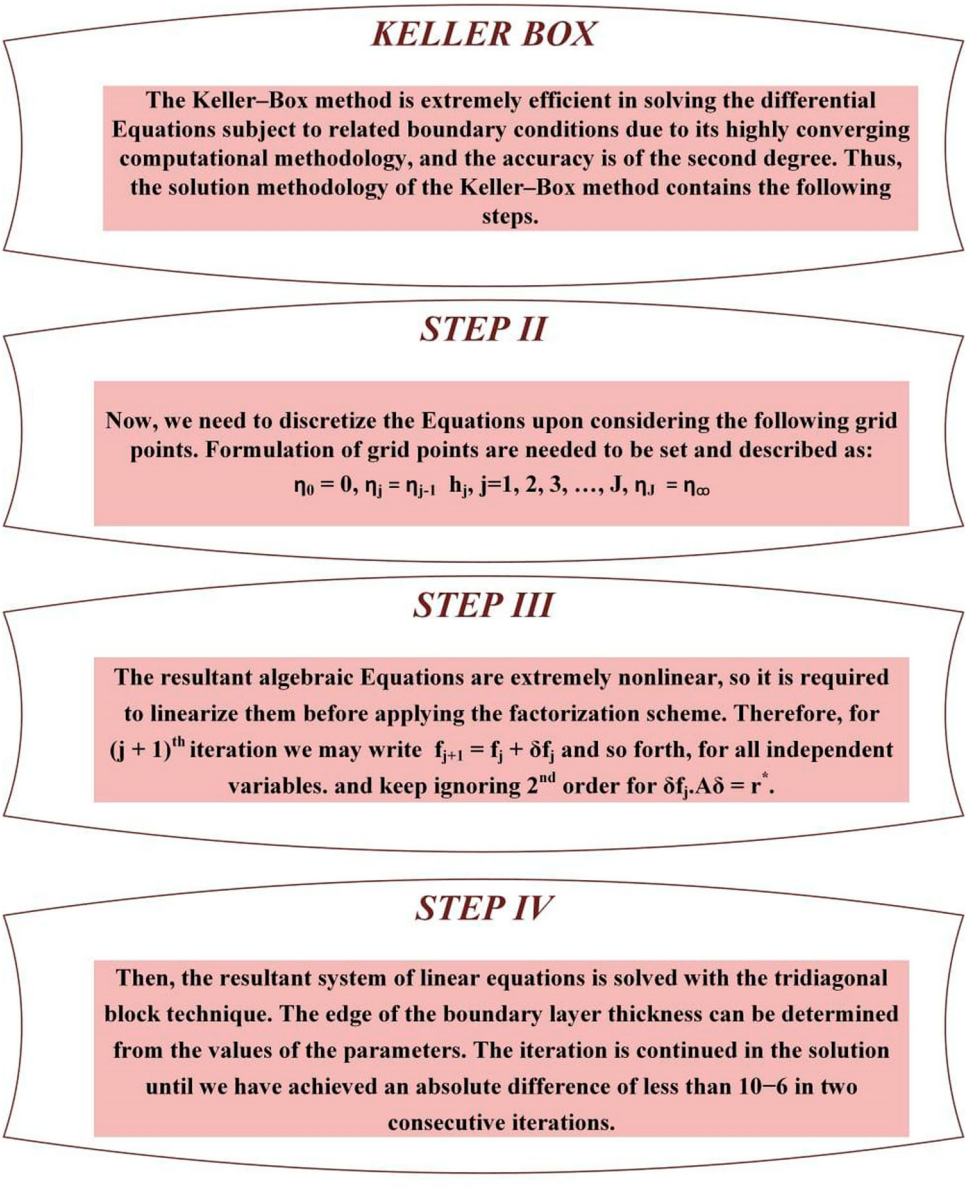


Chart 2. Complete demonstration for the Keller box.

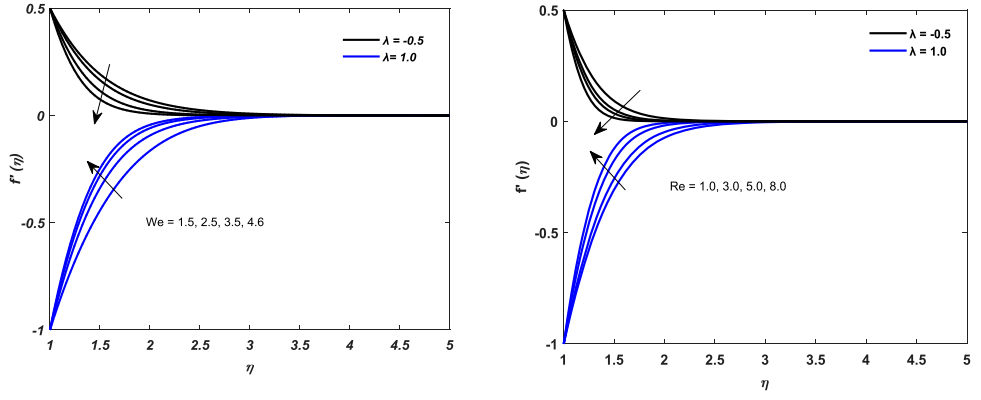


Figure 2. (a,b): Demonstration of We and Re of the velocity field for stretching and shrinking the cylinder.

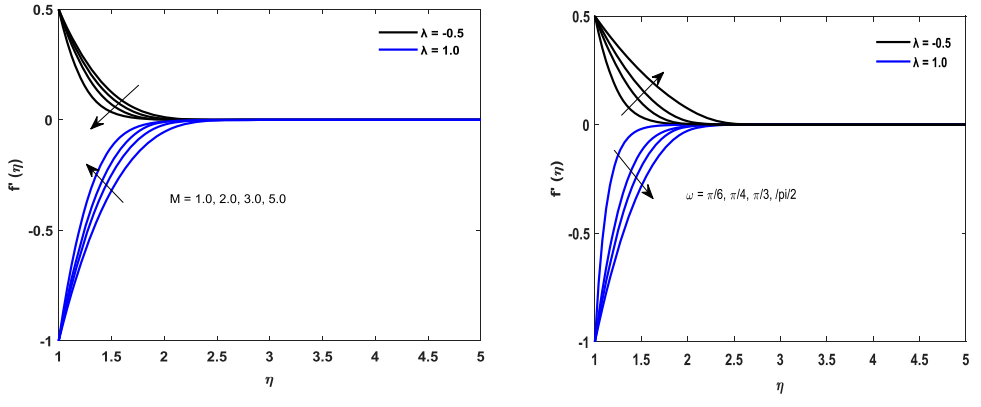


Figure 3. (a,b): Demonstration of M and ϖ of the velocity field for stretching and shrinking the cylinder.

5. Debate on numerical outcomes

This section presents the numerical outcomes obtained during the execution of the numerical technique. In this study, the execution of the inclination of a magnetic dipole, nanoscale exchange of heat with cylinder geometry is made to analyze the Cross Nanofluid. In the deep eye, it is observed that there is established a scheme for the solution of nonlinear higher order ODEs presented in Equations (13)–(15). Hence to reduce the order of ODEs the shooting scheme is applied. Furthermore, Bvp4c is applied to get the numerical simulations of the model. Weissenberg number is symbolized as *We*, Pr for Prandtl number, Schmidt ratio is described as Sc, local Reynold number is represented as Re, etc. Pictorial explanations for the numerical results are shown in Figures 2–8. Statistical data are collected to check the manner of all physical parameters and quantities from graphs (9–17). The parameter obtained from the ratio between initial velocity and free stream velocity is λ, if $\lambda < 0$ it is called the shrinking case and $\lambda > 0$ gives the stretching case.

Equation of momentum, which shows the velocity magnitude of Cross nanofluid, is attached with *We*, *n*, *Re*, *A*, *M*, and ϖ parameters. All necessary graphs and statistical pictures show the effects of the velocity of the Cross nanofluid.

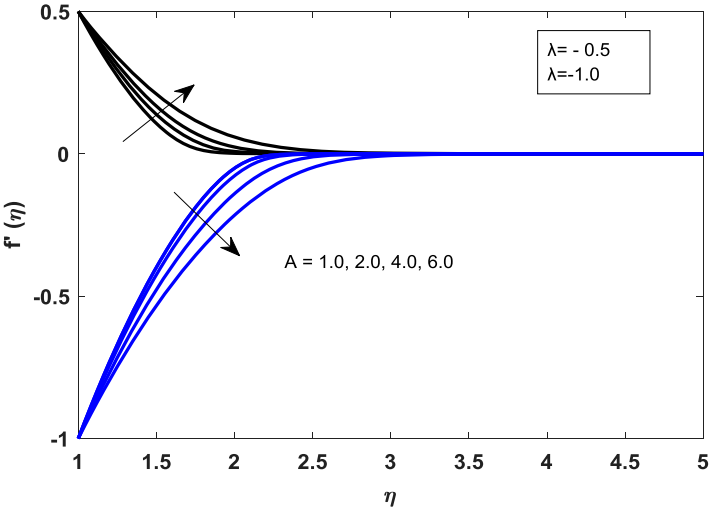


Figure 4. Demonstration of A of the velocity field for stretching and shrinking the cylinder.

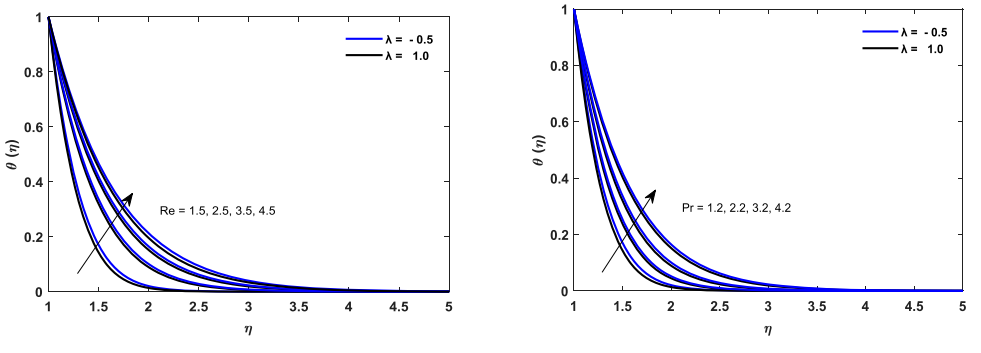


Figure 5. (a,b): Demonstration of Re and Pr of temperature field for stretching and shrinking the cylinder.

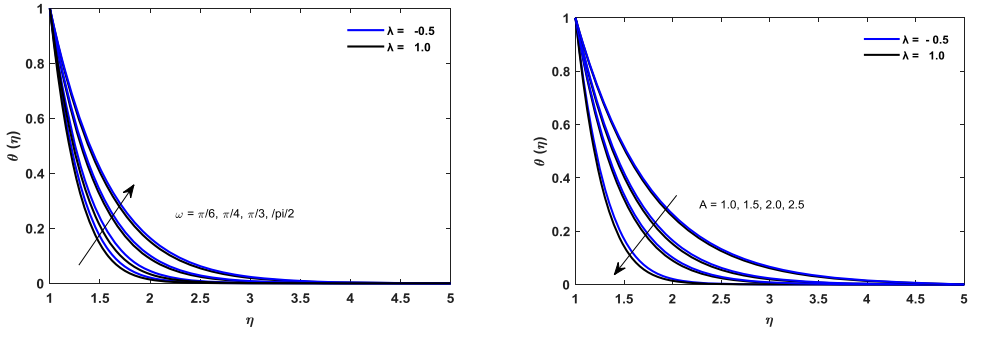


Figure 6. (a,b): Demonstration of ϖ and A of temperature field for stretching and shrinking the cylinder.

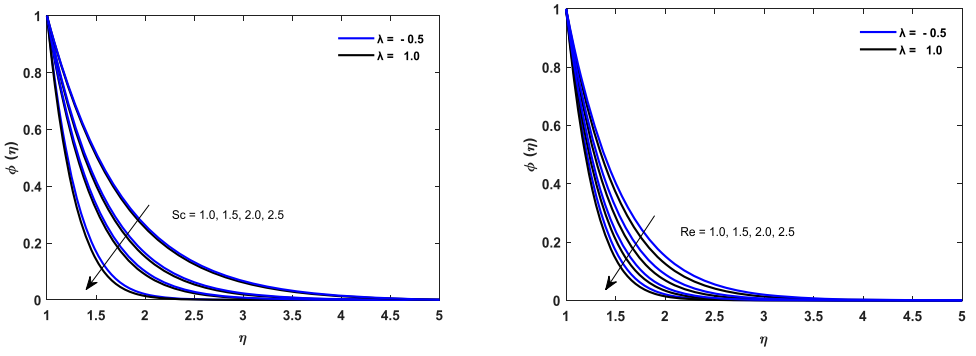


Figure 7. (a,b): Demonstration of Sc and Re of mass transfer field for stretching and shrinking the cylinder.

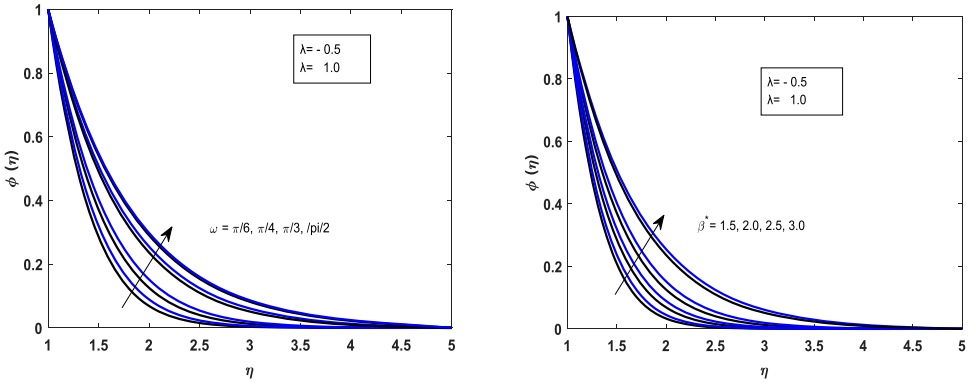


Figure 8. (a–b): Demonstration of ϖ and β^* of the velocity field for stretching/shrinking the cylinder.

When cylinder shrinks, then velocity becomes higher and due to relaxation of time factor, velocity field goes down for the shrinking case and gets higher for the stretching case. Velocity distribution gets fast for *Re* in the case of stretching and the opposite attitude is shown for the shrinking cylinder. Physically, Re is the ratio between viscous forces and inertial forces and greater Re results greater inertial forces. As in the case of stretching cylinder, inertial forces become lose and velocity gets fast speed while for the shrinking cylinder these forces become stronger and play an obstacle role in flow due to this flow, the velocity gets down. Demonstration of *We* and *Re* of velocity field for stretching and shrinking cylinder is shown in Figure 2(a,b). Figure 3(a,b) demonstrates the physical impact of *M* and ϖ of velocity field for stretching and shrinking cylinder. Both parameters produce Lorentz force and due to this, the velocity gets down. As Lorentz force is the combination of electric and magnetic force and they both retard the flow speed, hence the velocity is decreased. Figure 4 shows the demonstration of *A* of the velocity field for stretching and shrinking the cylinder. Figure 5(a,b) demonstrates the physical impact of *Re* and *Pr* on the temperature field for stretching and shrinking the cylinder. *Pr* increases the thermal thickness because this factor temperature of the Cross fluid is goes up for both cases. Figure 6(a,b) demonstrates ϖ and *A* of temperature field for stretching and shrinking the cylinder. The inclination angle reduces the velocity due to which the temperature of the Cross nanofluid increases rapidly.

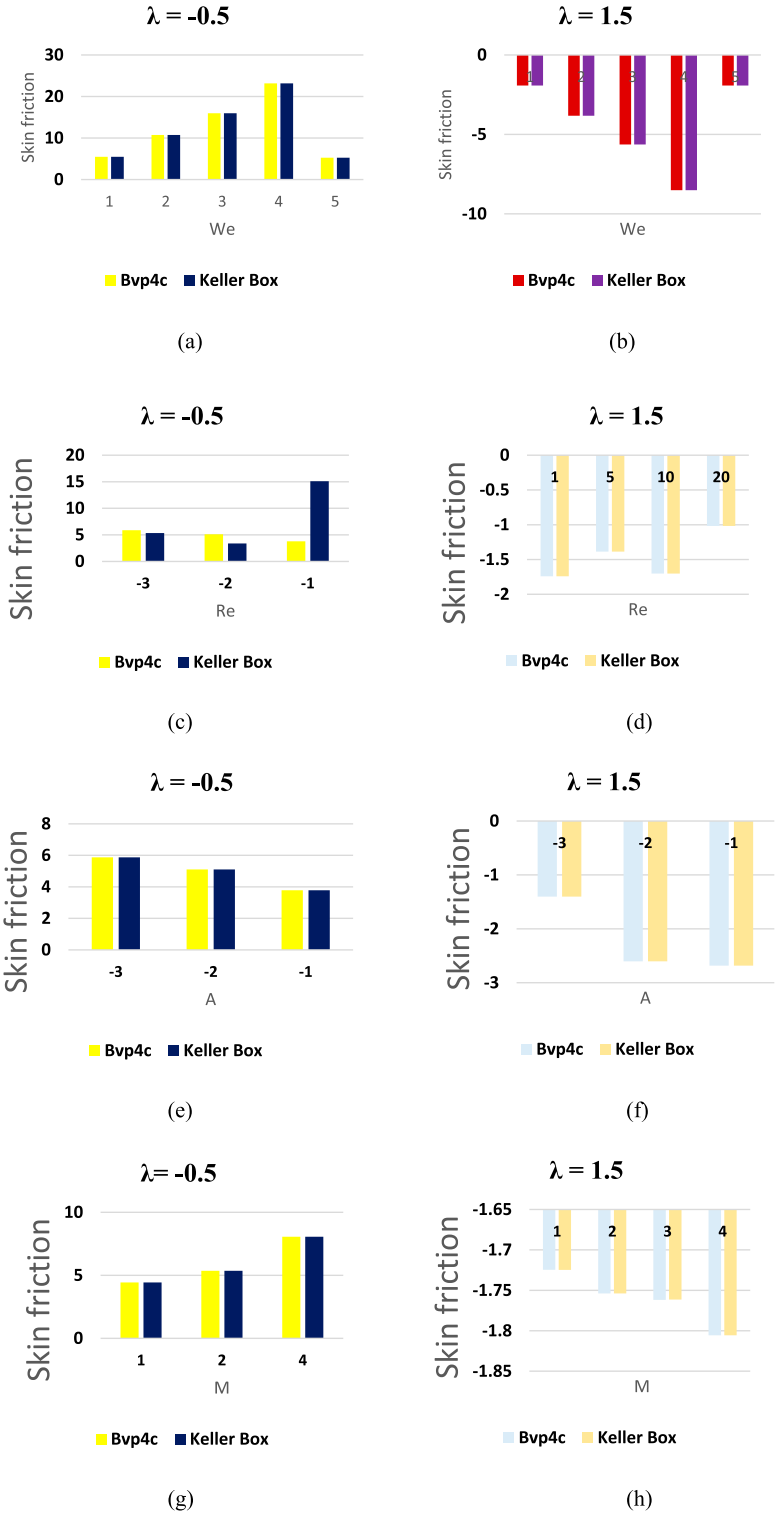
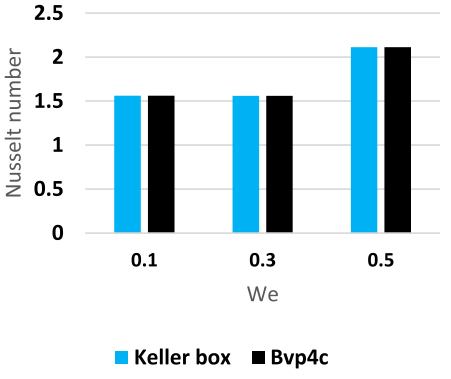
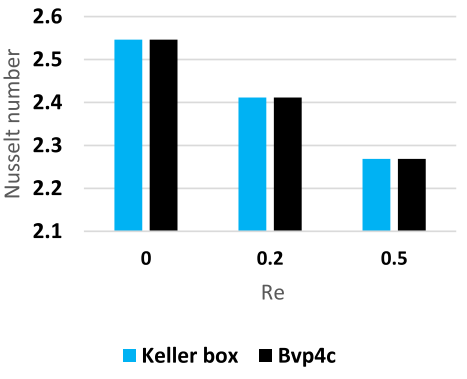


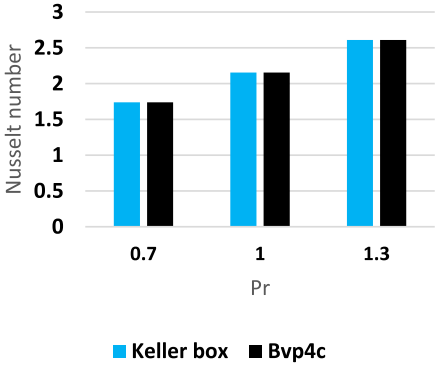
Figure 9. (a–h): Demonstration of skin friction for shrinking/stretching case for various physical parameters We, Re, A, and M.



(a)



(b)



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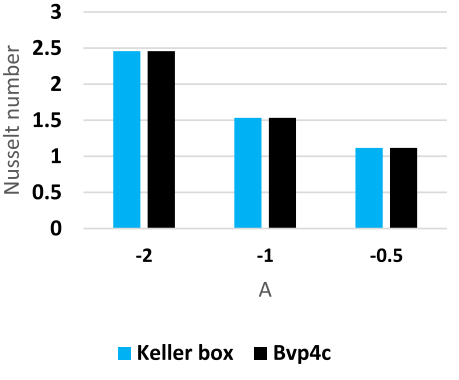

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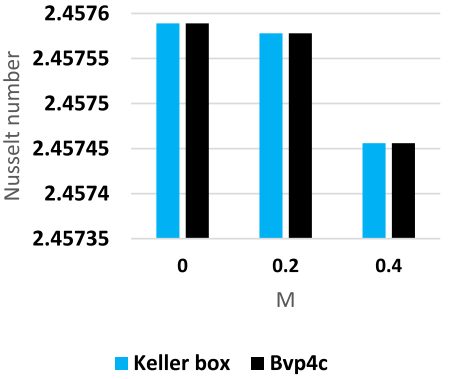

(e)

Figure 10. (a–e): Demonstration of We, Re, Pr, A, and M with the Nusselt number.

Figure 7(a,b) demonstrates the *Sc* and *Re* of the mass transfer field for stretching and shrinking the cylinder. In both cases, these physical parameters are responsible for lower transportation of mass, while Figure 8(a,b) shows the inclination angle and β^*cause grow in the mass transfer field for stretching and shrinking the cylinder. Graphs (9–17) show the physical parameters are linked with the physical quantities. Figures 9 and 10

6. Concluding remarks

The nanoscale heat transporting phenomenon of the fluid is explored by including the Lorentz force and variable thermal conductivity with the mathematical model of the cross model. The Cross fluid helps to investigate the problem in the shear thinning/thickening region. The numerical representations of the problem are provided via the BVP4C scheme.

The concluding remarks of this study are provided as follows:

1. Thermal conductivity enhances the nanoscale transport of energy.
2. Incline angle parameters produce the Lorentz force and due to which the velocity gets down.
3. Due to the relaxation of the time factor velocity field goes down for the shrinking case and higher for the stretching case for the 'We' physical parameter.
4. Velocity distribution gets rapid for *Re* in the case of stretching and the opposite attitude is shown for the shrinking cylinder.
5. The designed model is successfully solved using numerical procedures based on the BVP4C.
6. Pr increases thermal thickness due to which factor temperature of the Cross fluid goes up for both cases.
7. The inclination angle reduces the velocity due to which factor temperature of the Cross nanofluid increases rapidly.

Disclosure statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

ORCID

Mohamed R. Ali http://orcid.org/0000-0002-0795-0709

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